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A CONCEPT OF SLOPE DESIGN OF EARTH DAM EMBANKMENT

by C. Y. Li

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A CONCEPT OF SLOPE DESIGN OF

EARTH DAM EMBANKMENT

C. Y. Li, Esq.
Civil Engineer, Gannett Fleming Corddry and Carpenter, Inc.
Harrisburg, Pennsylvania

SYNOPSIS

In the stability design of earth embankment, it is still a frequent practice to choose a constant slope from the crest down to the foundation. For embankment made of homogeneous cohesive soil this is not necessary, and as its height increases, becomes uneconomical. It may be proved that for cohesive soil embankment the slope profile is a curve with continually decreasing slopes from top down. In practice, the logical design for earth fill is to use a number of simple slopes to fit in the theoretical curved profile. The significance of varying slope design versus constant slope design lies in the saving of material used without reducing the factor of safety of the structure. The idea of varying slope design is not new and it can be found that numerous existing earth dams possess a number of simple slopes instead of one constant slope. However, it is the intention of this paper to discuss the basic principle of and to approximate a guide to a more logical and practical varying slope design for earth dam embankment and levees.

INTRODUCTION

For an embankment made of ideal cohesionless material such as dry clean sand, any constant slope with an inclination angle equal to or less than its angle of internal friction is stable regardless of height. The factor of safety against sliding failure will be:

$$F_S = \frac{\tan \phi}{\tan i}$$

where ϕ = angle of internal friction of sand

and i = angle of inclination of slope.

A constant slope for this case may therefore be stated as an ideal slope because the factor of safety is the same throughout the height.

In practice, engineers are dealing mostly with cohesive soil. It is an obvious fact that, for a given soil, the higher the fill the flatter must be its slopes in order to be stable. In other words, from the crest down, the slope decreases as the height increases, or the slope profile is a function of its height. Frontard[1], based on Coulomb's equation of shearing resistance and Rankine's hypothesis of conjugate stresses, derived mathematically that for fills made

1. P. J. Frontard "Surfaces de Glissement et calculs de stabilite' des massifs en terre a profil curviligne" Travaux, Juin 1948.

of cohesive material the stable slope, regardless of height, is a curve which can be expressed by equations and that the critical sliding surface is a plane. However, in the following determination of slope profile the basic assumption of the Swedish circular arc method of slope stability analysis whereby the failure surface is to be of cylindrical shape will be used.

Factor of Safety Against Sliding Failure Along Constant Slope

The factor of safety against sliding failure may be defined as the ratio of the total average shearing strength along the critical sliding surface to the total average shearing stress along the same surface.

Passing through any point along the slope of an embankment an infinite number of circular surfaces may be drawn. The surface with the minimum factor of safety would be the most critical one passing through the point investigated. If a sufficient number of points along the slope are analyzed for their critical sliding surface, the variation of the factor of safety with height can be determined. Fig. 1 shows the approximate relationship of factors of safety with height measured from the crest down of a 1 on 2 1/2 constant slope of homogeneous soil without seepage. It is assumed for convenience that the ratio of unit cohesion to unit weight is unity. It can be seen that, for constant slope, the factor of safety is constantly increasing from the toe upward. In other words, the sliding surface passing through the toe controls the design, and the constant slope provided above the toe is stronger than necessary for stability.

Critical Slope Profile

Let it be defined that the critical slope profile is the slope profile which is in the condition of incipient failure everywhere along the slope from the crest down. In Fig. 2a assume aopb is a section of a critical slope consisting of cohesive soil with unit weight π, unit cohesion c and angle of internal friction $\emptyset$, of which ao is the crest and OX, OY are rectangular coordinate axes with origin at o.

At any point p on the slope, it may be written $y = f\left(x, \emptyset, \frac{c}{\pi}\right)$ ....(1); for a soil of known values of $\emptyset$, c and π, Equation 1 becomes $y = f(x)$ ....(2). Equation 2 can be plotted by cut and trial using the Swedish cylindrical sliding surface method of analysis.

Starting with an arbitrary small height h_1 from the crest, trial cylindrical surfaces for various trial angle i_1 are analyzed until the condition for critical slope whereby the rupture force equals the resisting force is reached. The trial angle i_1 which satisfies this condition determines the critical slope for h_1; thus h_1 and i_1 serve to locate point 1 on the critical slope profile. Next, for an arbitrary height h_2 from the crest, which is slightly greater than h_1, repeat the same process with various trial angle i_2 connected with point 1. The angle i_2 for incipient failure and h_2 determines point 2 on the critical slope profile. Continue down in the same manner to locate points 3, 4 ... until a sufficient number of points are obtained to enable the plotting of a smooth curve which is the desired critical slope profile as expressed by Equation 2.

Fig. 2b shows an illustration of the graphical solution of one of the trial surfaces. For a given $\emptyset$, and making $\frac{c}{\pi} = 1$, any trial surface may be investigated for stability regardless of system of units used.

Fig. 3 is the approximate critical slope profile with no seepage for ϕ values of 5^o, 15^o and 25^o. The curve profile is dimensionless with its coordinates as the multiple of the ratio of c and π or expressed in $\frac{1}{Ns\,\pi}$, where Ns is the Taylor's stability number.[2]

Example of Preliminary Design

Assume it is required to make a preliminary design of slopes of an earth dam 100 feet high to be built of homogeneous soil with a shearing strength $c = 750$ lbs. per sq. ft., $\phi = 22^o$ and $\pi = 125$ lbs. per cu. ft.

By allowing a factor of safety of 1.5, the available shearing strength will be $c = \frac{900}{1.5} = 600$ lbs. per sq. ft. $\tan \phi = \frac{0.402}{1.5} = 0.268 = \tan 15^o$. Since $\frac{c}{\pi} = \frac{600}{125} = 4.8$ feet. The slope profile required for $\phi = 15^o$ may be obtained from Fig. 3 by increasing the values of the coordinate axes 4.8 times to convert to feet unit. Based upon the obtained theoretical slope profile, a few simple slopes may be fitted in. Detail analysis at many points along the slopes are then made by taking into consideration other important factors such as seepage force, neutral pressure and pore pressure. If the final slopes have to be flattened, it may be done by following the general trend of the original theoretical slopes.

Practical Significance

Figs. 4a and 4b show an example of the results of analyses of an earth dam of homogeneous material with different slopes. The critical condition for downstream slopes is considered when the reservoir is full, and for upstream slopes when complete drawdown. Flow nets are drawn to determine neutral pressures. For the slopes investigated, the critical sliding surfaces pass through the toe or foundation. The curves with factor of safety 1.5 for different slopes investigated are shown for comparison.

It is seen that whether one slope is safer than the other depends on the relative values of c and ϕ. In this particular example, with $c = 0.6$ tons per sq. in. $\tan \phi = 0.51$, for the downstream side (Fig. 4a), slope 1 has the same factor of safety as slope 2. The adopting of slope 1 means a saving of 51.4 cubic yards of material per foot length of dam. For a dam with a crest of 1000 feet, this amounts to a total saving of 51,400 cubic yards, or \$36,000 at a unit price of \$0.70 per yard. For the upstream slope (Fig. 4b), the adopting of slope 1, as compared with slope 2, reduces the total yardage by 176,700, or the total cost by \$123,700 at \$0.70 per yard, and in the meantime increases the factor of safety slightly. The use of slope 3, as compared with slope 1, increases the factor of safety from 1.5 to 1.7 besides saving 289,000 cubic yards of material or \$202,300.

The practical significance of the varying slopes compared with the constant slope in earth embankment design depends obviously on the height and length of dam, the type of soil to be used and its availability and foundation conditions. In most cases, with fill of moderate or great height, the total saving, although only a small percentage of the total cost of the whole structure, is appreciable by itself.

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2. D. W. Taylor "Fundamentals of Soil Mechanics" p. 456, John Wiley & Sons, Inc. (1948).

Conclusion

In the final design of earth dams a system of factors such as seepage, pore pressure, heterogeneity of materials, foundation uncertainty, construction method, tension cracks, etc. tend to complicate the problem. In many cases, it is very difficult to assign values of c and Ø obtained by laboratory testing in the Coulomb's empirical equation which will represent the actual condition to a reasonable extent, or many sets of values of c and Ø must be used for investigation. Nevertheless, it is reasonable to believe that the theoretical critical slope profile as derived here is an approach to a logical design because its basis of derivation, considering the gravitational action of the dead weight of the material, plays a primary role for the stability of slopes.

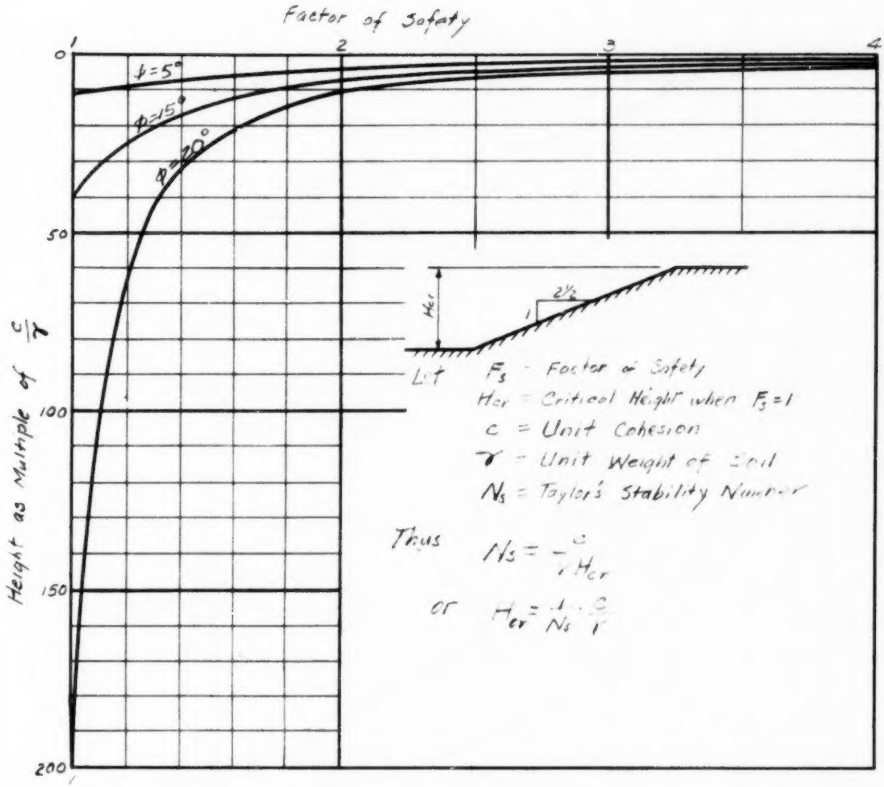

Fig. 1

Factor of Safety vs. Height of a 2-1/2:1 Constant Slope Embankment

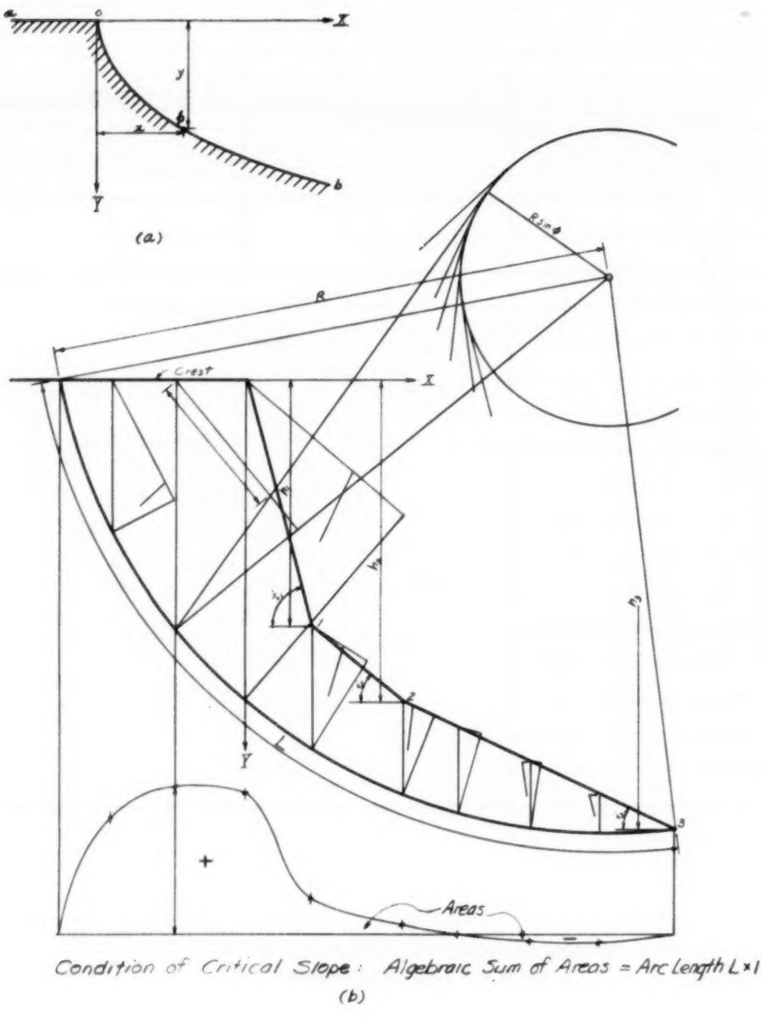

Fig. 2

Graphical Solution of Critical Slope Profile

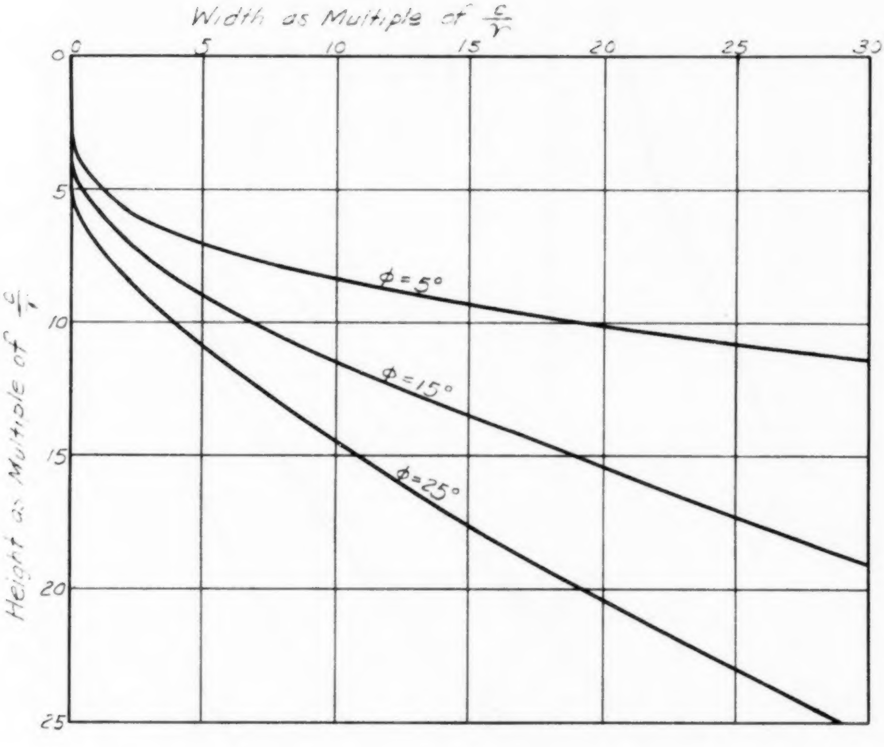

Fig. 3

Critical Slope Profile for Homogeneous Cohesive Soil Without Seepage

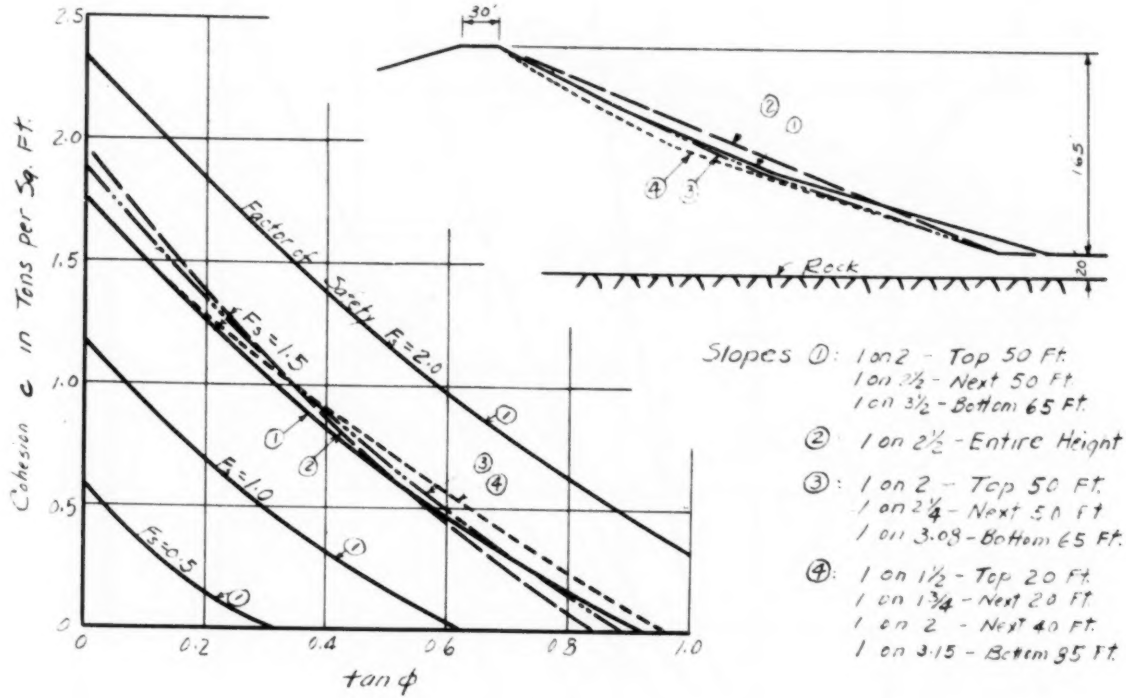

Fig. 4a

Example of Result of Stability Analysis - Downstream Slope

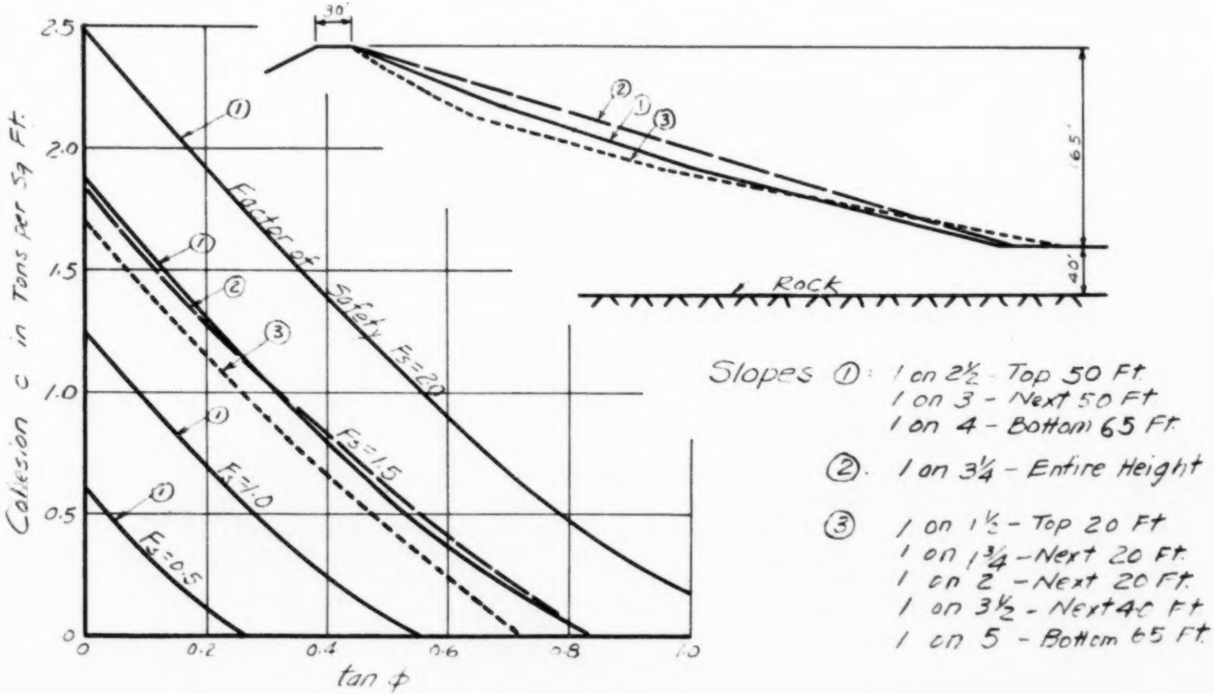

Fig. 4b

Example of Result of Stability Analysis - Upstream Slope